

## Problem 1.15

[Difficulty: 5]

**1.15** For Problem 1.14, the initial horizontal speed of the sky diver is 70 m/s. As she falls, the  $k$  value for the vertical drag remains as before, but the value for horizontal motion is  $k = 0.05 \text{ N} \cdot \text{s}/\text{m}^2$ . Compute and plot the 2D trajectory of the sky diver.

**Given:** Data on sky diver:  $M = 70 \cdot \text{kg}$   $k_{\text{vert}} = 0.25 \cdot \frac{\text{N} \cdot \text{s}^2}{\text{m}^2}$   $k_{\text{horiz}} = 0.05 \cdot \frac{\text{N} \cdot \text{s}^2}{\text{m}^2}$   $U_0 = 70 \cdot \frac{\text{m}}{\text{s}}$

**Find:** Plot of trajectory.

**Solution:** Use given data; integrate equation of motion by separating variables.

Treat the sky diver as a system; apply Newton's 2nd law in horizontal and vertical directions:

Vertical: Newton's 2nd law for the sky diver (mass  $M$ ) is (ignoring buoyancy effects):  $M \cdot \frac{dV}{dt} = M \cdot g - k_{\text{vert}} \cdot V^2$  (1)

For  $V(t)$  we need to integrate (1) with respect to  $t$ :

Separating variables and integrating: 
$$\int_0^V \frac{V}{\frac{M \cdot g}{k_{\text{vert}}} - V^2} dV = \int_0^t 1 dt$$

so 
$$t = \frac{1}{2} \cdot \sqrt{\frac{M}{k_{\text{vert}} \cdot g}} \cdot \ln \left( \frac{\sqrt{\frac{M \cdot g}{k_{\text{vert}}}} + V}{\sqrt{\frac{M \cdot g}{k_{\text{vert}}}} - V} \right)$$

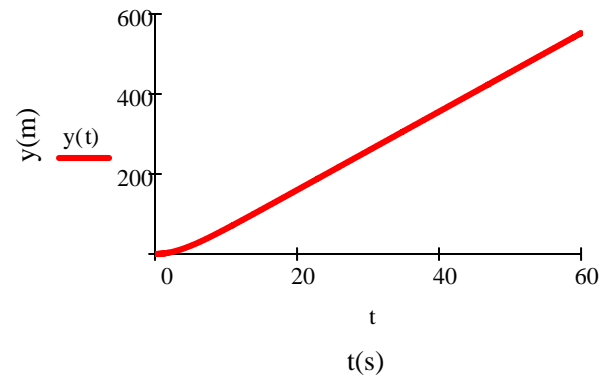
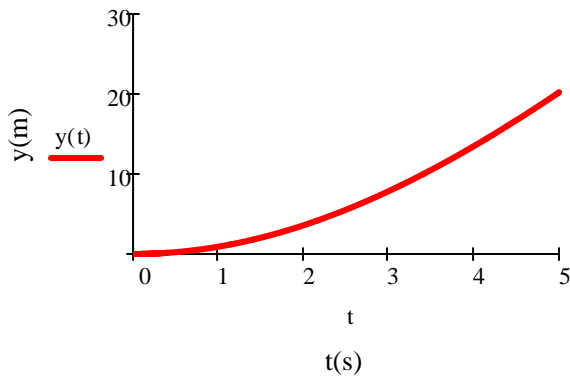
Rearranging 
$$V(t) = \sqrt{\frac{M \cdot g}{k_{\text{vert}}}} \cdot \frac{\left( e^{2 \cdot \sqrt{\frac{k_{\text{vert}} \cdot g}{M}} \cdot t} - 1 \right)}{\left( e^{2 \cdot \sqrt{\frac{k_{\text{vert}} \cdot g}{M}} \cdot t} + 1 \right)}$$
 so 
$$V(t) = \sqrt{\frac{M \cdot g}{k_{\text{vert}}}} \cdot \tanh \left( \sqrt{\frac{k_{\text{vert}} \cdot g}{M}} \cdot t \right)$$

For  $y(t)$  we need to integrate again:  $\frac{dy}{dt} = V$  or  $y = \int V dt$

$$y(t) = \int_0^t V(t) dt = \int_0^t \sqrt{\frac{M \cdot g}{k_{\text{vert}}}} \cdot \tanh \left( \sqrt{\frac{k_{\text{vert}} \cdot g}{M}} \cdot t \right) dt = \sqrt{\frac{M \cdot g}{k_{\text{vert}}}} \cdot \ln \left( \cosh \left( \sqrt{\frac{k_{\text{vert}} \cdot g}{M}} \cdot t \right) \right)$$

$$y(t) = \sqrt{\frac{M \cdot g}{k_{\text{vert}}}} \cdot \ln \left( \cosh \left( \sqrt{\frac{k_{\text{vert}} \cdot g}{M}} \cdot t \right) \right)$$

After the first few seconds we reach steady state:



Horizontal: Newton's 2nd law for the sky diver (mass  $M$ ) is:

$$M \cdot \frac{dU}{dt} = -k_{\text{horiz}} \cdot U^2 \quad (2)$$

For  $U(t)$  we need to integrate (2) with respect to  $t$ :

Separating variables and integrating: 
$$\int_{U_0}^U \frac{1}{U^2} dU = \int_0^t -\frac{k_{\text{horiz}}}{M} dt \quad \text{so} \quad -\frac{k_{\text{horiz}}}{M} \cdot t = -\frac{1}{U} + \frac{1}{U_0}$$

Rearranging

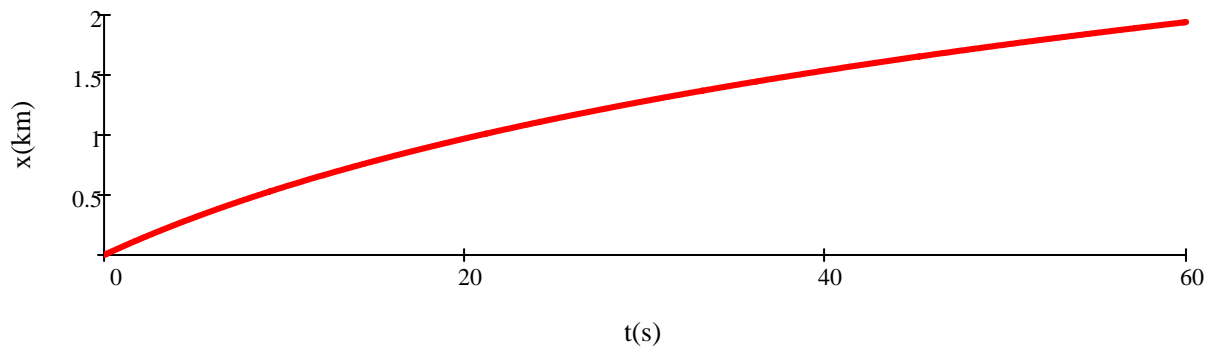
$$U(t) = \frac{U_0}{1 + \frac{k_{\text{horiz}} \cdot U_0}{M} \cdot t}$$

For  $x(t)$  we need to integrate again:

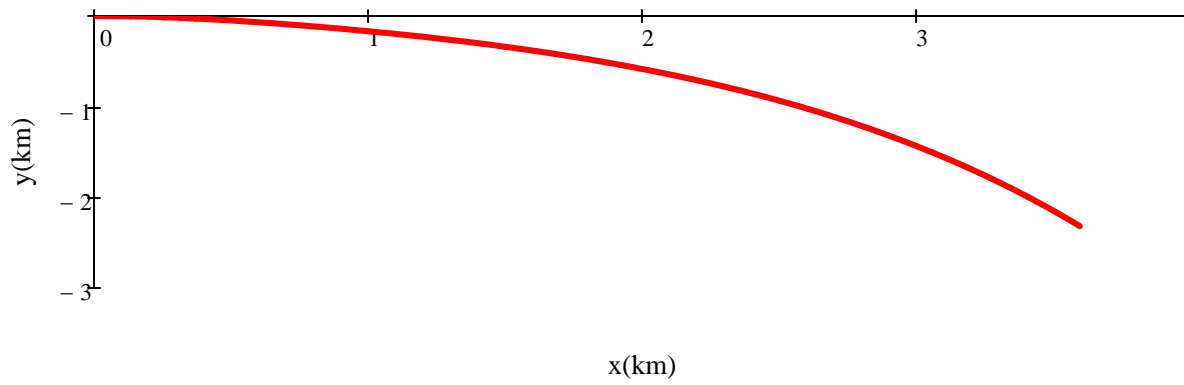
$$\frac{dx}{dt} = U \quad \text{or} \quad x = \int U dt$$

$$x(t) = \int_0^t U(t) dt = \int_0^t \frac{U_0}{1 + \frac{k_{\text{horiz}} \cdot U_0}{M} \cdot t} dt = \frac{M}{k_{\text{horiz}}} \cdot \ln \left( \frac{k_{\text{horiz}} \cdot U_0}{M} \cdot t + 1 \right)$$

$$x(t) = \frac{M}{k_{\text{horiz}}} \cdot \ln \left( \frac{k_{\text{horiz}} \cdot U_0}{M} \cdot t + 1 \right)$$



Plotting the trajectory:



These plots can also be done in Excel.